Computationally Efficient Approach to Simulating Collisionless Stopping of Relativistic Beams in Plasmas

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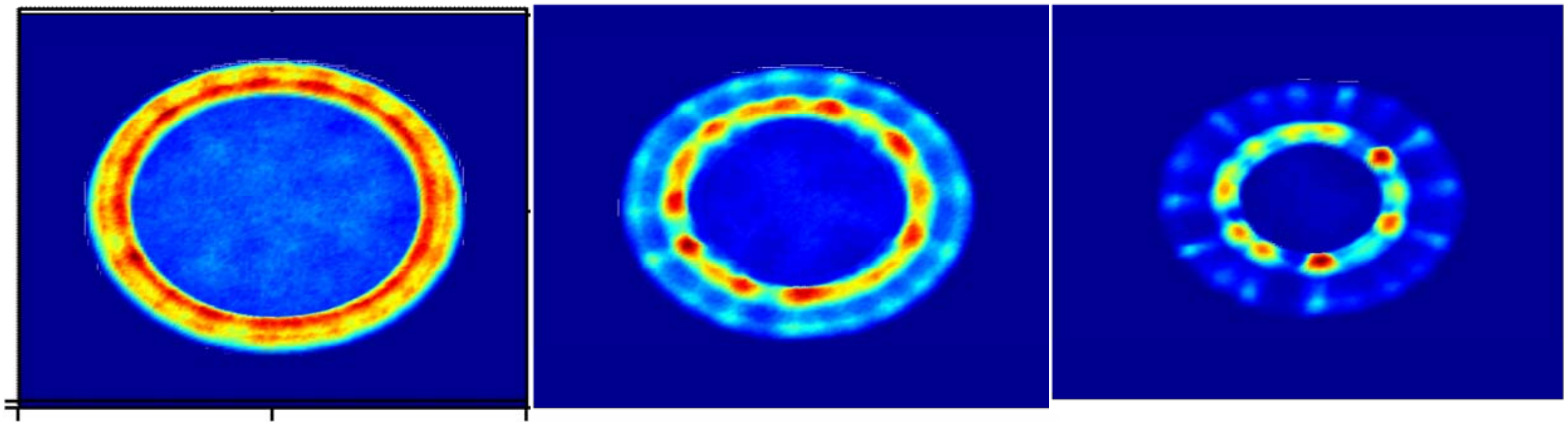
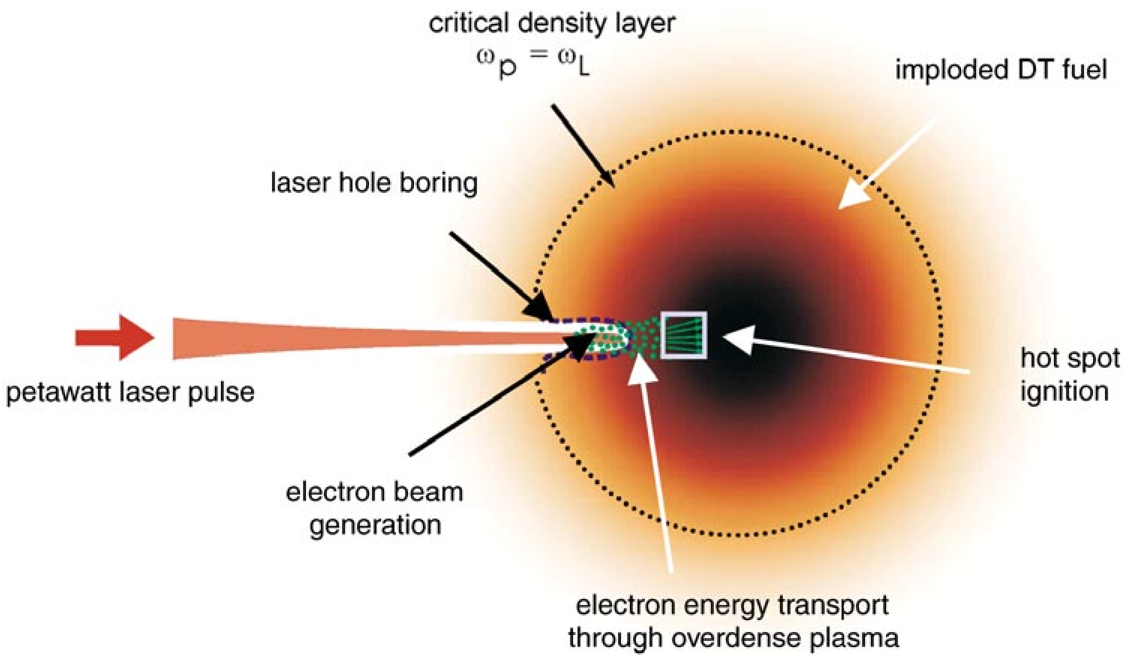


With: O.Polomarov (UT), I. Kaganovich, and A. Sefkow (PPPL)

Rice Astrophysics Workshop, May 15, 2007

Physics Issues in Fast Ignition



E-beam stoppage was considered to be trivial in the original papers → collisional stopping of 1-2 MeV beams. What if the energy is much higher?

Why collective instabilities?

- **High ignitor pulse energy (100 kJ?) → high current**

$$U = \gamma mc^2 I\tau / e = 100 \text{ kJ}$$

If τ is measured in picoseconds: **$I = 2 \times 10^{11}$ A/($\gamma\,\tau$)**

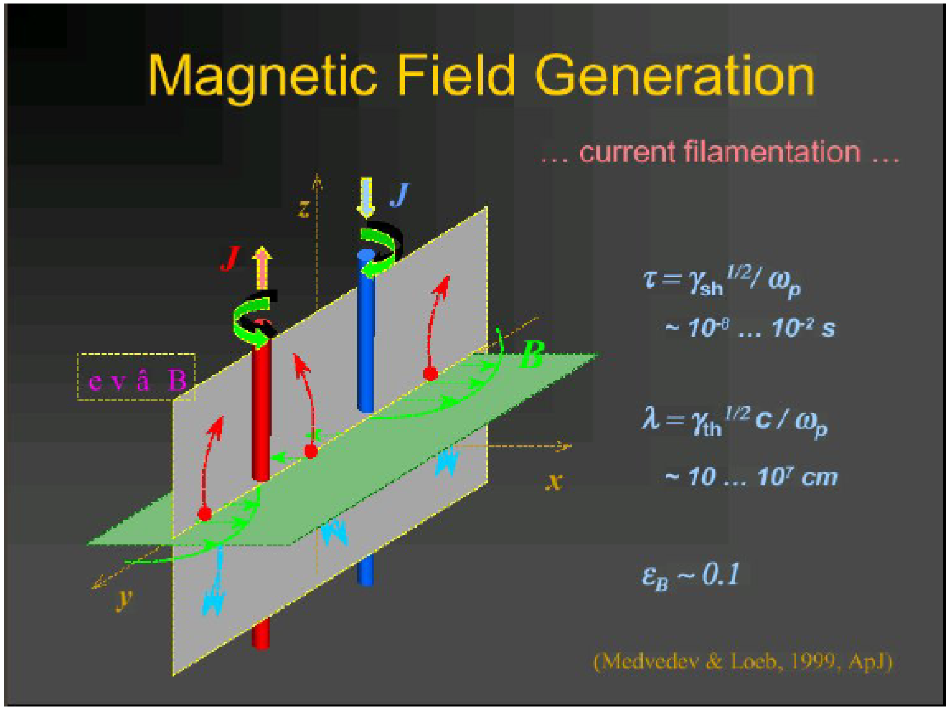
- **Currents exceeding the limiting Alfven current $I_A = \gamma mc^3/e = 17\,\gamma$ kA undergo catastrophic filamentation known as Weibel instability → possible collective mechanism of extracting energy from the beam into B-fields and fast ions**

- **Increasing γ reduces I → enough electrons in the corona to ignite the target**

New features of $\gamma >> 1$ and $n_b/n_p << 1$ (relativistic beam in the core) regime

- Collisional stopping is insufficient for stopping within alpha range (electrons shoot through target) → have to rely on anomalous (collective) instabilities such as Weibel
- Growth rate of Weibel instability also slows down: $\gamma_w << \omega_p$ → have to invent a computational tool which is different from standard PIC simulations which resolve $\Delta t = 1/\omega_p$ time scale
- Fast electrons with $\gamma >> 1$ and ambient plasma electrons behave as two different species with different masses

Relevance to Astrophysics



Global questions:

- **How are magnetic fields produced in afterglow shocks of GRB sources?**
- **What is the efficiency and can equipartition be achieved with relativistic beams?**
- **For how long do B-fields persist?**

Note: astrophysicists and fast ignitors want the same: rapid equipartition of relativistic beam and plasma energies!

Basic Physics Questions

- **What is the final nonlinear outcome (after thousands of plasma oscillations!) of the Weibel instability?**
- **How is the reconnection of current filaments in completely collisionless plasma takes place (more relevant to astrophysics than to Fast Ignition)**
- **What is the interplay between Weibel and tearing instabilities? Is the out-of-plane magnetic field dynamically important, or is it simply a by-product driven by the in-plane B-field?**

Existing Computational Approaches

- **PIC:**

  L. O. Silva et al., Phys Plasmas **10**, 1979 (2003),

  M. Honda et al., Phys Plasmas **7**, 1302, (2000).

  C. Ren *et al.*, Phys. Rev. Lett. **93**, 185004 (2004)

  Advances all plasma and beam particles

- **Hybrid modeling:**

  LSP (hybrid mode),

  T. Taguchi et al., Phys. Rev. Lett., **86**, 5055 (2001).

  Solves hydrodynamic equation for background plasma

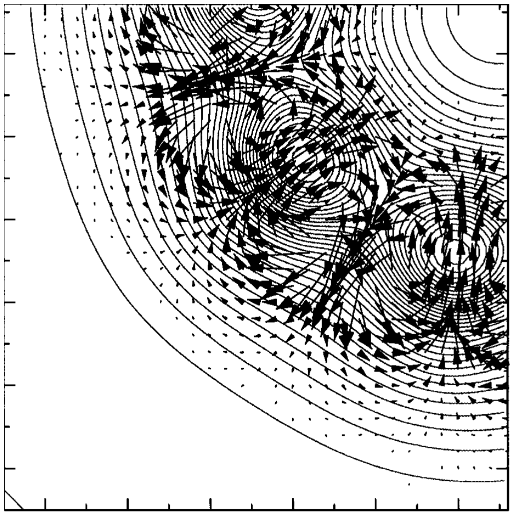
- **Resistive Codes:**

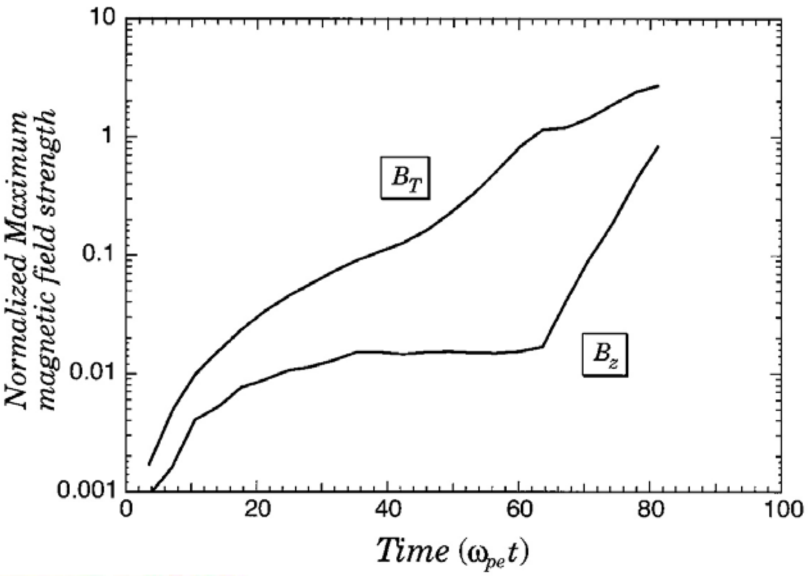
  Gremillet et. al., Phys. Plasmas 3, 941 (2002).

  Simplest form of Ohm's Law → *valid for highly collisional plasmas*

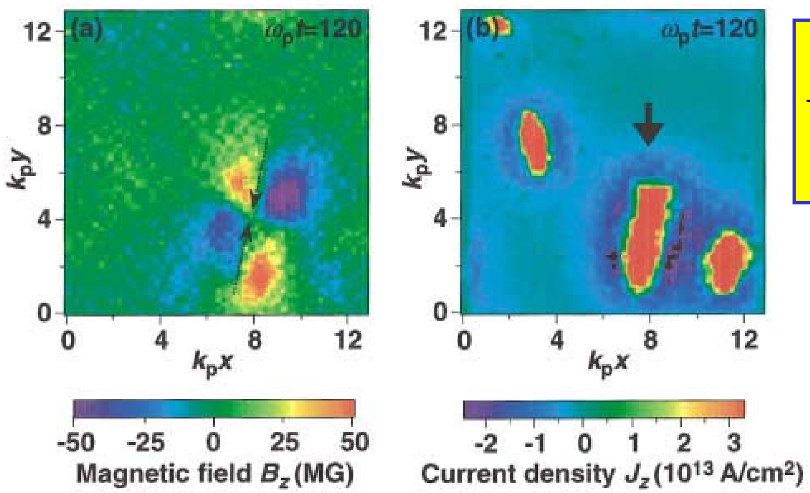
<u>**First two → Computationally expensive because they resolve electron plasma frequency.**</u>

Importance of Tearing Instability and Out-of-Plane B-field





Taguchi et.al., PRL’01



$$\vec{v}_\perp = -\frac{c}{4\pi en}\vec{e}_z \times \vec{\nabla} B_z$$

Honda et.al., PRL’00

New modeling for $n_b << n_p$ (weak hot component), $\gamma >> 1$ (relativistic), collisionless

Solution: develop a code that
(i) treats beam electrons kinetically

(ii) assumes quasi-neutrality and does not solve equations of motion for ambient plasma

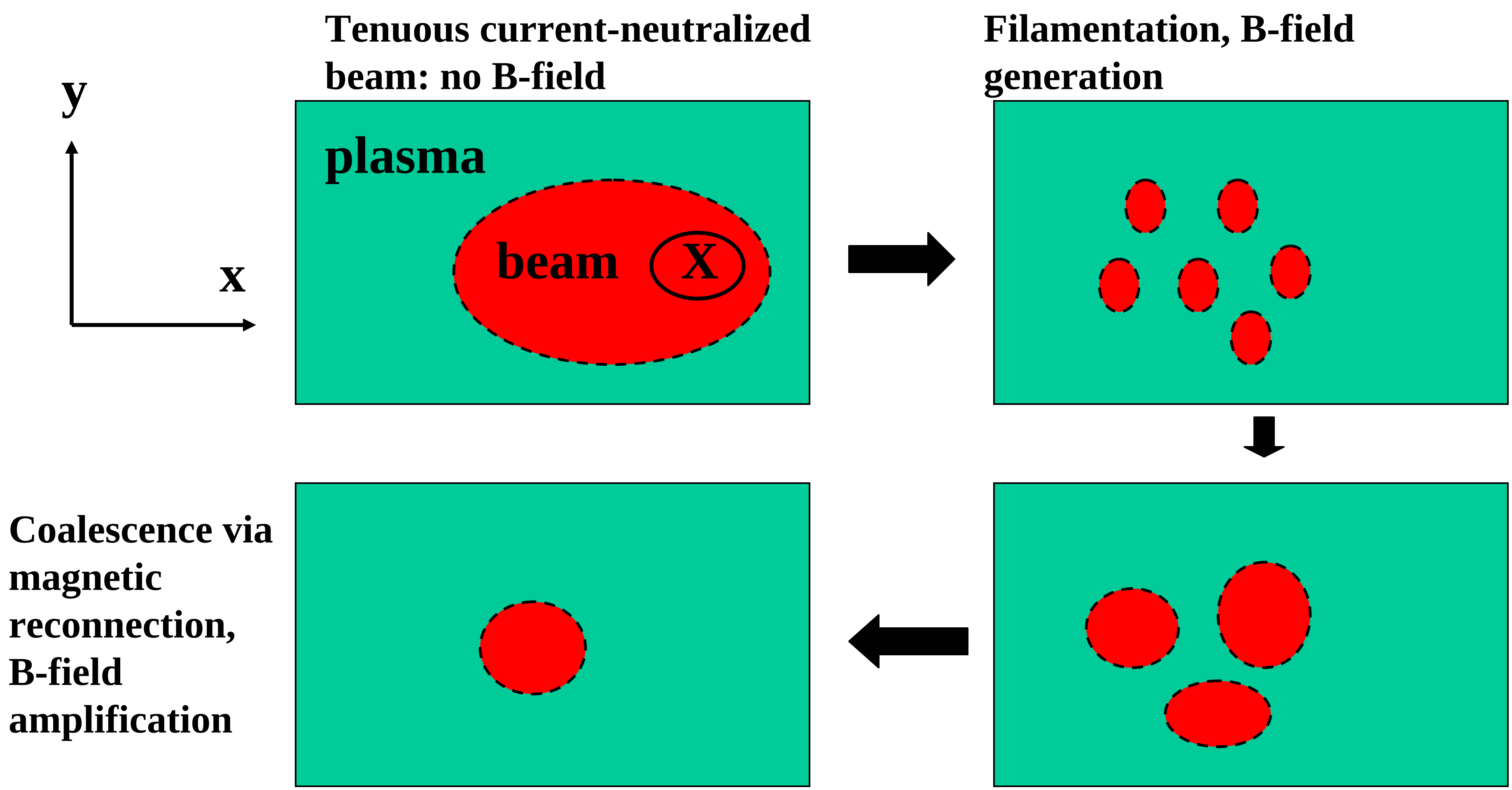
(iii) does not resolve ambient electron plasma frequency

(iv) correctly models merger of filaments and evolves electron beam into a strongly nonlinear regime

- Present version of the code is two-dimensional, but extendable to 3-D
- Question to workshop participants: are there any astrophysical situations to which such density asymmetry is relevant??

Hybrid simulation approach: fluid ambient plasma and kinetic fast electrons

•Two dimensional simulation: (x,y) computational domain. E-beam propagates in z-direction



Major Assumptions

- Quasi-neutrality because evolution is slow compared with $\Delta t = 1/\omega_p$: $n_b + n_e = n_{e0}$
- Neglect electrostatic two-stream instability modes with finite k_z because they saturate and don't deplete the energy of relativistic electron beam
- Neglect plasma collisions (can be put in at a later stage)
- Restricted to two dimensions (will be extended to three later on) for simplicity

Cold electron dynamics

$$\boxed{\frac{\partial \vec{v}_e}{\partial t}+\left(\vec{v}_e\cdot\vec{\nabla}\right)\vec{v}_e=-\frac{e}{m}\left(\vec{E}+\frac{\vec{v}_e\times\vec{B}}{c}\right)}$$

Conservation of generalized vorticity:

$$\boxed{\frac{\partial \vec{\Omega}}{\partial t}-\vec{\nabla}\times\vec{v}_e\times\vec{\Omega}=0}$$

where

$$\boxed{\vec{\Omega}=\vec{\nabla}\times\vec{v}_e-e\vec{B}/mc}$$

- **For initially cold collisionless plasma Ω = 0 for all times!**

- **In 2-D:** $$\boxed{\vec{B}=\vec{e}_z B_z-\vec{e}_z\times\vec{\nabla}\psi}$$ **where $\psi = A_z$**

- **After simple algebra from Ω = 0 →** $\mathbf{v_{ez} = \psi/mc}$

Field Equation for ψ and B_z

$$\boxed{\left(\nabla^2 - \frac{\omega_{pe}^2 + \omega_{pe}^2 \left\langle \gamma_j^{-1} \right\rangle}{c^2}\right)\psi = \frac{4\pi \vec{e}_z \cdot \vec{J}_b}{c}}$$

Neglect electron inertia: $\frac{e\psi}{mc^2} \approx -\frac{n_b}{n_e}$ **→ flux "frozen" into beam**

$$\boxed{\left(\nabla^2 - \vec{\nabla}\ln(n_e)\cdot\vec{\nabla} - \frac{\omega_{pe}^2}{c^2}\right)B_z = -\frac{4\pi n_e}{c}\vec{e}_z \cdot \vec{\nabla}\times\left(\frac{\vec{J}_{b\perp}}{n_e}\right)}$$

Note: out-of-plane magnetic field is only generated if J_b/n_e has a non-vanishing curl → electron inertia is essential. This effect is known in MHD literature as whistler-driven reconnection. Whistler-driven reconnection requires an extra Hall term in the MHD equations.

Extension to 3-D

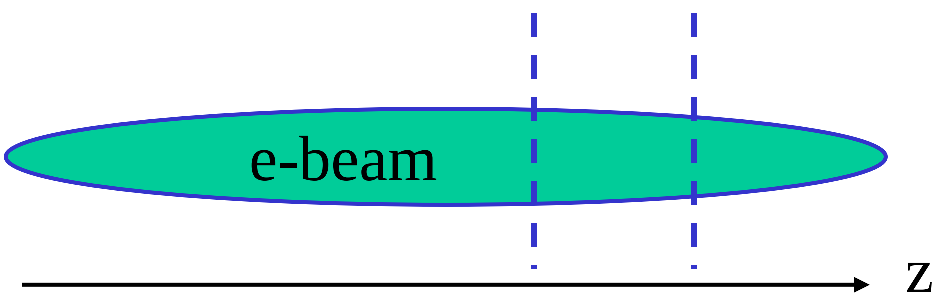
$$\vec{B} = \vec{e}_z B_z(\vec{x}_\perp, \lambda z) - \vec{e}_z \times \vec{\nabla}_\perp \psi(\vec{x}_\perp, \lambda z) + \vec{\nabla}_\perp \phi(\vec{x}_\perp, \lambda z)$$

small — dominant — smaller

- **For $\lambda << 1$ neglect B_z and ϕ → slice e-beam and solve for ψ:**

$$\left(\nabla_\perp^2 - k_{pe}^2\right)\psi = 4\pi \vec{e}_z \cdot \vec{J}_b / c$$

$$E_z = -\frac{1}{c}\frac{\partial \psi}{\partial t}$$



$$\vec{E}_\perp \approx -\frac{e}{2mc^2}\vec{\nabla}\psi^2$$

Particle and Field Equations

- **Equation of motion for beam particles:**

$$\frac{d(\gamma_j v_{j\perp})}{dt} = -\frac{e p_{jz}}{m\gamma_j}\vec{\nabla}\psi + \frac{e^2}{2m^2c^2}\vec{\nabla}\psi^2 + \vec{F}_\perp$$

where

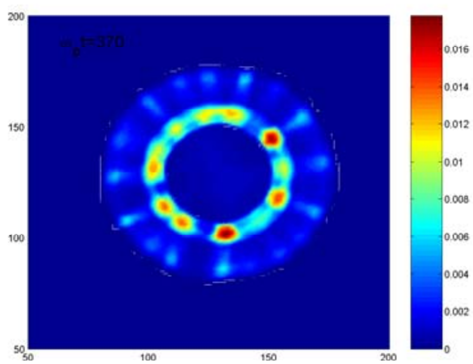
$$\vec{B} = \vec{e}_z B_z - \vec{e}_z \times \vec{\nabla}\psi$$

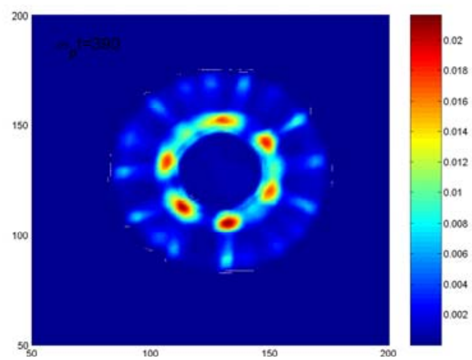
- **Field Equation (solved using a multigrid algorithm):**

$$\left(\nabla^2 - \frac{\omega_{pe}^2}{c^2}\right)\psi = \frac{4\pi\vec{e}_z\cdot\vec{J}_b}{c}$$

and

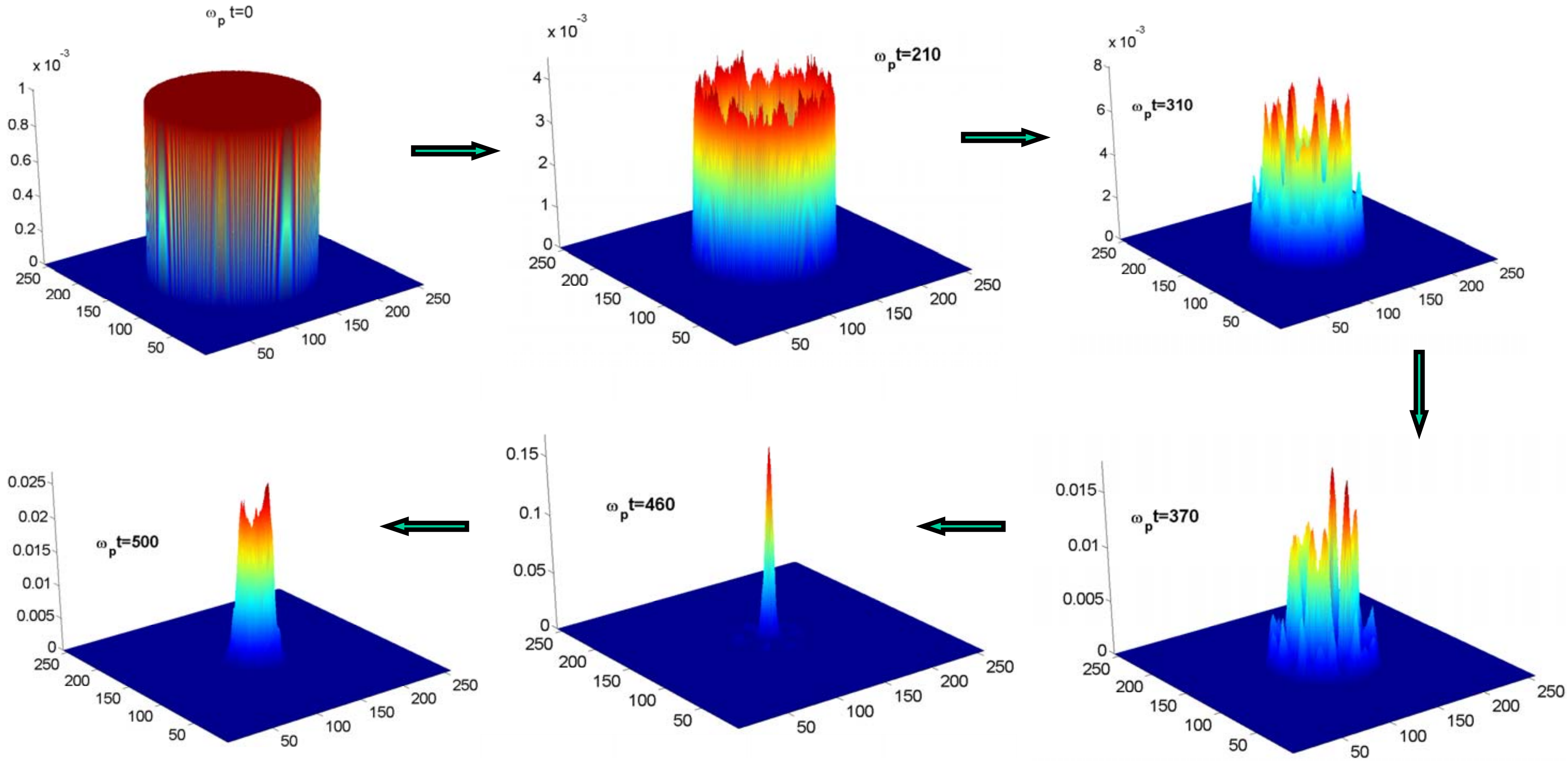
$$\left(\nabla^2 - \vec{\nabla}\ln(n_e)\cdot\vec{\nabla} - \frac{\omega_{pe}^2}{c^2}\right)B_z = -\frac{4\pi n_e}{c}\vec{e}_z\cdot\vec{\nabla}\times\left(\frac{\vec{J}_b}{n_e}\right)$$



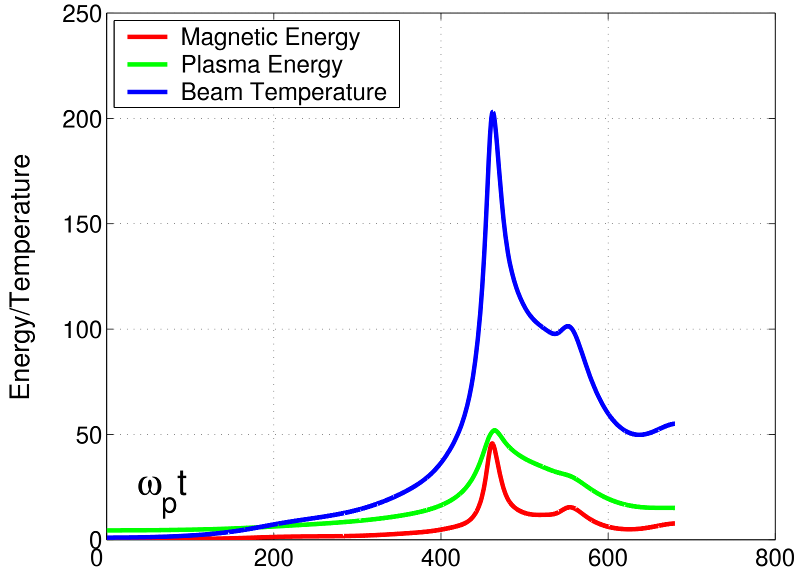


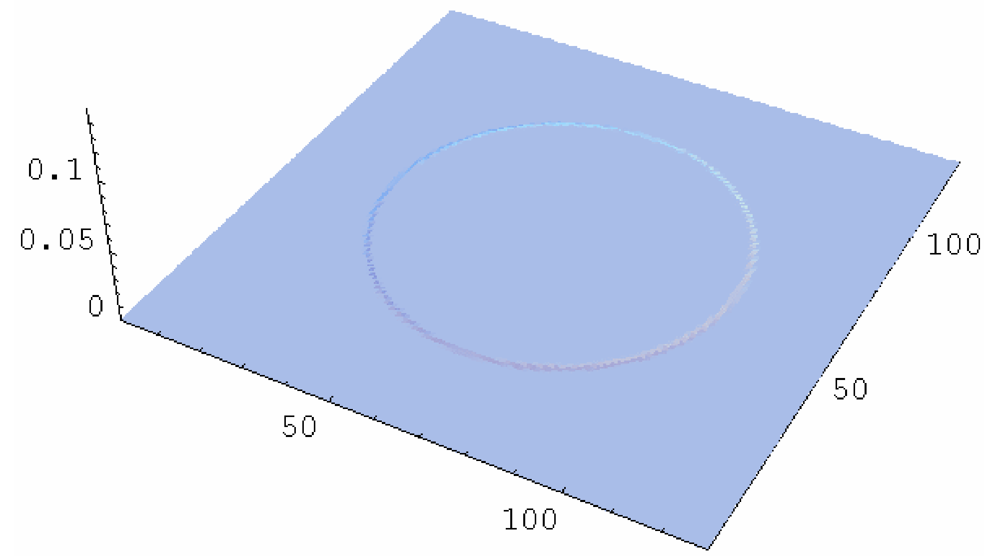
Example:
filaments merger
for $n_p/n_b = 1000$

Weibel instability of a beam in a strongly overdense plasma ($I < I_A$)



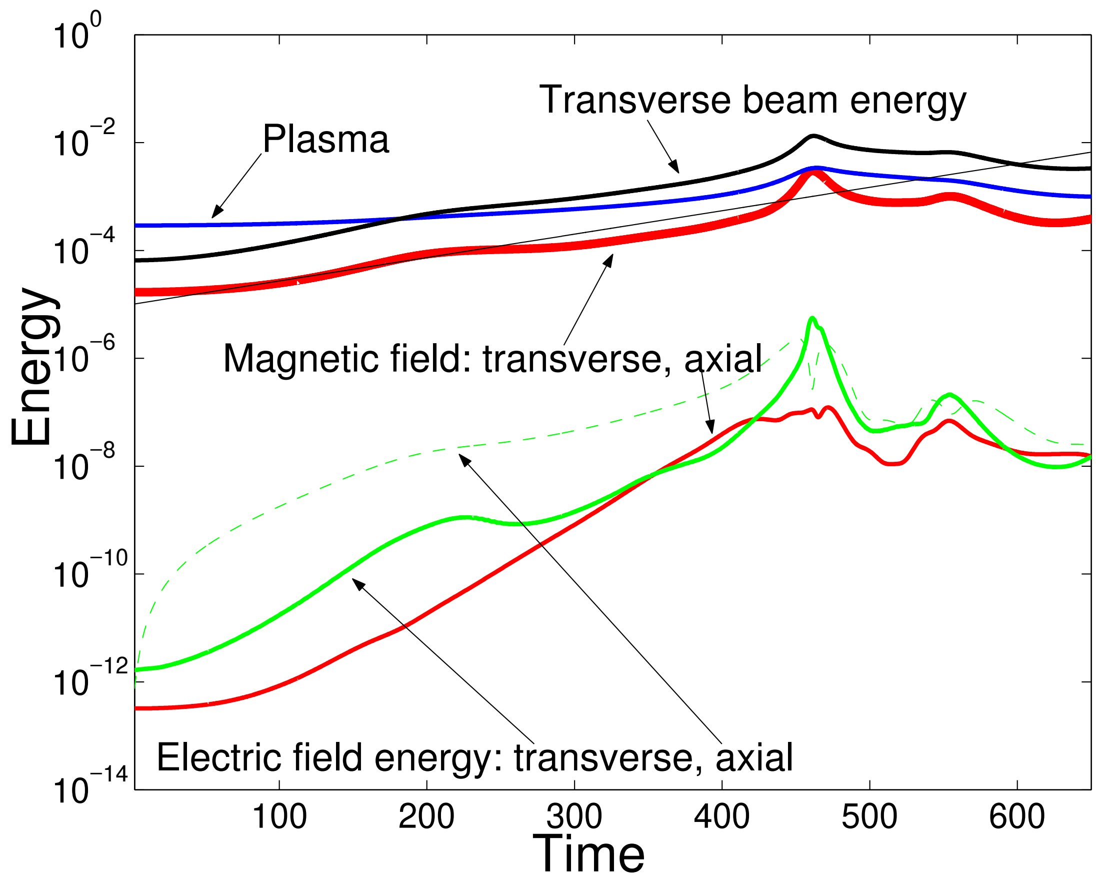
First results: plasma heating and magnetic field generation





Weibel (filamentation) instability of a relativistic electron beam with diameter. Simulation box is 256x256 (or), $2x10^6$ particles. Peak density compression of the beam: 150 times. Energies are normalized to the relativistic beam temperature

Energy deposition into plasma



Instability saturates when

$$R_b \sim \frac{v_\perp}{\gamma_\omega} = \frac{v_{bz}}{\gamma_\omega} \times \frac{\Omega_c}{\gamma_\omega}$$

$$\text{But} \quad R_b \sim \frac{c}{\omega_{pe}}$$

In agreement with

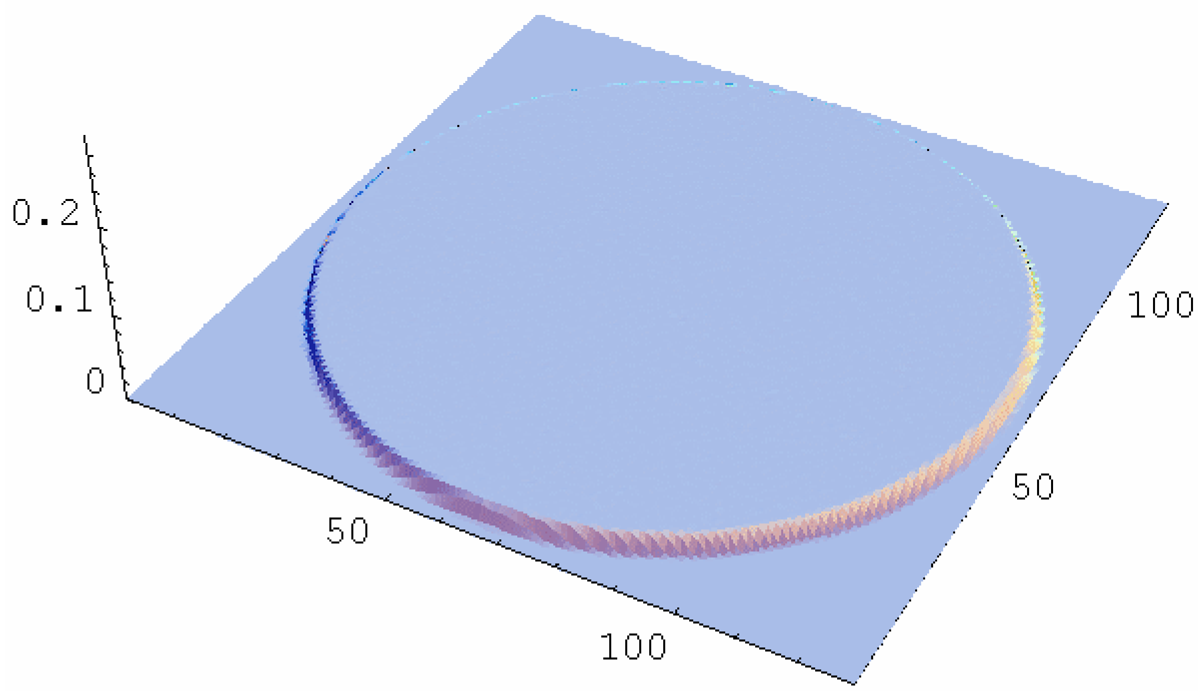
$$\gamma_\omega \sim \omega_{bounce}$$

R. C. Davidson et al., Phys. Fluids **15**, 317 (1972).

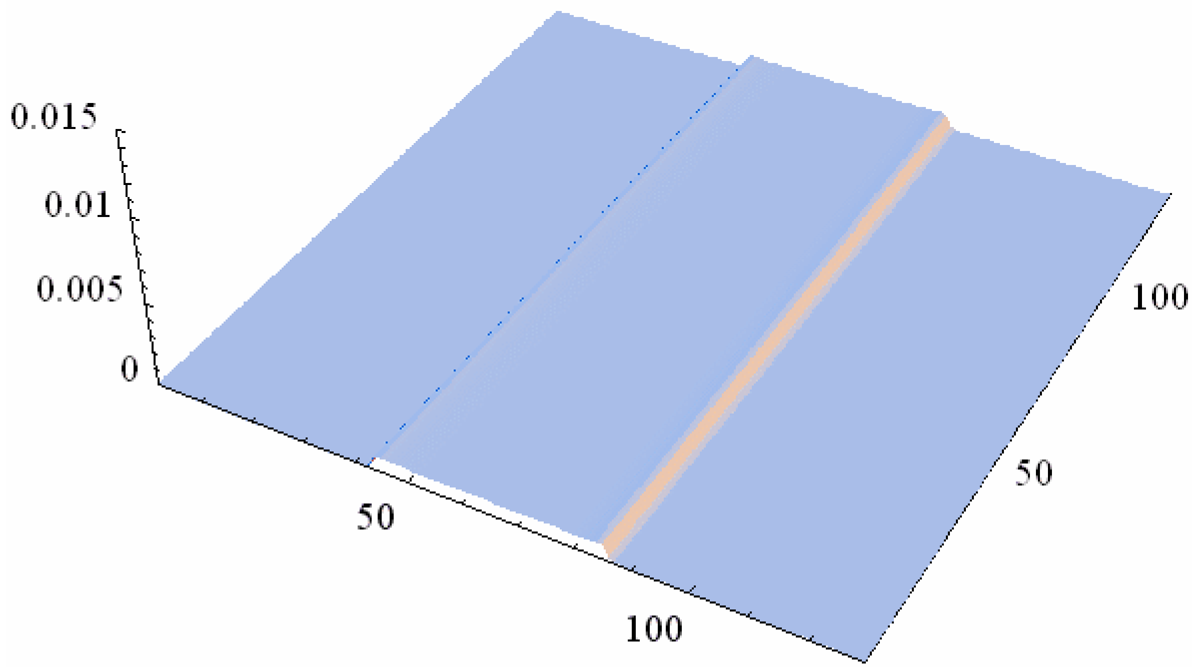
For low-current beams

$$\frac{B^2/8\pi}{(\gamma-1)n_b mc^2} \propto \frac{\gamma}{2(\gamma-1)} \frac{I}{I_A}$$

Weibel instability of high current beam



Weibel Instability is dimensionality-dependent



Benchmarking with the LSP PIC