HOMEWORK PLACEMARKER
PHYS 516, Fall 2017

Don’t get antsy, just wait until next year!

**Due date:** Whenever!

**But just in case you are bored in the interim:** Let us pick up a thread encountered this past semester. The Legendre functions $Q_n(z)$ of the second kind can be defined (for $z > 1$ or $z < -1$) in terms of the normal Legendre polynomials $P_n(t)$ via the integral representation (due to Neumann)

$$Q_n(z) = \frac{1}{2} \int_{-1}^{1} \frac{P_n(t)}{z-t} \, dt .$$

(1)

Here $n$ is an integer.

(a) Use this to show that the $Q_n(z)$ functions must assume the form

$$Q_n(z) = \frac{1}{2} P_n(z) \log \left| \frac{z+1}{z-1} \right| + \text{polynomial} ,$$

(2)

and obtain a closed form for the polynomial.

(b) By presuming analytic continuation of this form to the interval $|z| < 1$, show that the orthogonality of the $P_n$ on the interval $[-1, 1]$ does not automatically imply similar orthogonality of the $Q_n$ on the same interval.

However, exploit parity properties to ascertain a restricted orthogonality property for the the $Q_n$ on this interval. Be sure to address the integral convergence issue.